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ANALYSIS OF WASTE-HEAT THERMOELECTRIC POWER GENERATORS

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Abstract—A real thermoelectric power generator utilizing waste heat is proposed. The generator is treated as an external and internal irreversible heat engine. The specific power output of the generator is analyzed and compared with that of the Carnot, endoreversible and external reversible thermoelectric heat engines.

Keywords—Thermoelectric generator, waste heat, finite-time thermodynamics.

INTRODUCTION

The usefulness of comparing real processes with the corresponding reversible ones, which takes an infinite time to complete, appears questionable in classical thermodynamics. Reversible limits are not close enough to real performances to be useful in guiding the improvements of processes. These remarks have led many authors to the examination of whether it is possible to find more realistic limits to the performance of actual processes. In particular, asking how well systems can perform if they are to deliver power, not just work, has led to investigations in both fundamental thermodynamics and in many engineering applications. This investigation field is called finite-time thermodynamics. A literature survey of the finite-time heat engine is given by Wu [1]. Most of these finite-time thermodynamic analyses have concentrated on the performance of endoreversible heat engines. Only a recent paper by Gordon [2] has investigated how the additional introduction of frictional and heat leak losses affect heat engine behavior. An endoreversible heat engine is defined as an internal reversible but external irreversible heat engine which exchanges heat with its surroundings through a finite temperature difference as shown in Fig. 1.

However, the influence of heat transfer is not the only generic source of irreversibility on the power performance of a waste-heat thermoelectric generator. The generator has other easily identifiable internal irreversibilities in Joulean heating and thermal conduction heat flow. This paper will take another step in finite-time thermodynamics to develop an analytical expression for the specific power output of a real waste-heat thermoelectric heat engine by considering both the internal and external irreversibilities.

SPECIFIC POWER OF AN IRREVERSIBLE WASTE-HEAT THERMOELECTRIC GENERATOR

The art and science of thermoelectric energy conversion has gradually evolved to a high level of performance over the past few decades. A novel thermoelectric power generator utilizing waste heat is proposed in this paper. The waste-heat thermoelectric generator is a much simpler system than a conventional vapor heat engine. The thermoelectic generator does not use any working fluid and therefore requires no boiler, condenser, working fluid pump or turbine. These components of the conventional power plants are replaced by thermoelectric power modules, which are simply compact heat exchangers integrated with thermoelectric generators. A thermoelectric generator is the preferred method for producing electric power directly from waste heat in specific missions where reliability and maintenance-free operation are essential. Also, for small-scale low temperature waste heat sources, the steam turbine cycle is not very efficient and economical. Thermoelectric generators can compete favorably with the conventional vapor waste-heat cycles.

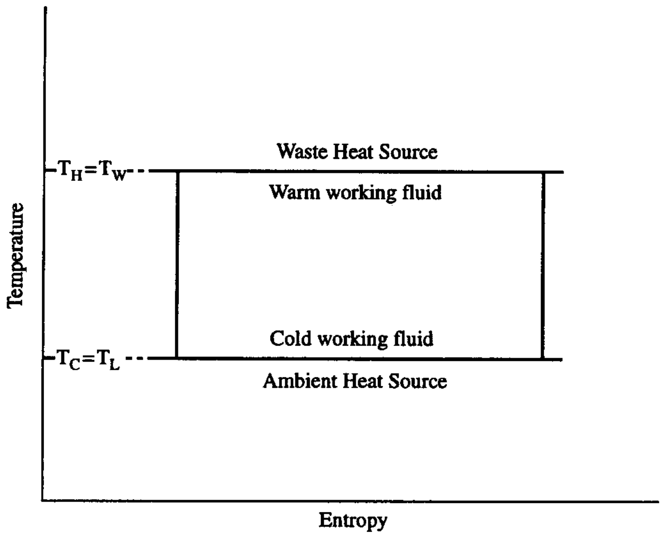

Fig. 1. Endoreversible waste-heat heat engine.

The waste-heat thermoelectric generator is illustrated in Fig. 2. Two dissimilar materials, n- and p- semi-conductors, are connected by metal conductors, making good thermal and electrical contact at the hot and cold junctions. The hot junction is heated by the waste-heat source with a heat exchanger and the cold junction is cooled by the ambient heat sink with another heat exchanger. A temperature gradient across the thermoelectric material drives electron charge carriers from the hot to the cold junction side and produces a voltage. The electrical energy generated from the thermal energy does work in an external circuit; the remainder of the thermal energy is rejected from the cold junction. The power output depends upon the temperature difference, the properties of the semi-conductor materials and the external load resistance (or electric current).

Consider the waste-heat thermoelectric generator to be a heat engine, as shown in Fig. 3. Heat is transferred from the warm waste-heat source at T_H to the hot junction at T_W and from the cold junction at T_C to the ambient heat sink at T_L. The junctions are assumed to be isothermal and to have negligible resistances to flow of heat and electricity compared with resistances of legs n and p. The legs have constant rectangular cross-sections, which are not necessarily equal. It is also assumed that the resistivity (ρ), the thermal conductivity (k) and the Seebeck coefficient (α) for

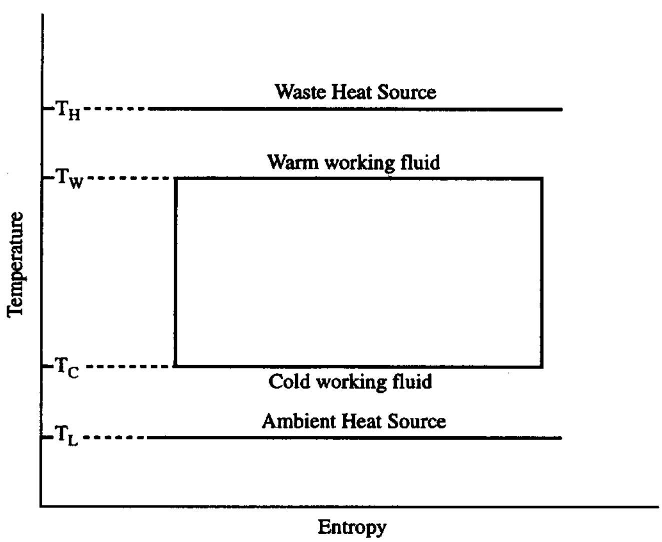

Fig. 2. Real waste-heat thermoelectric generator.

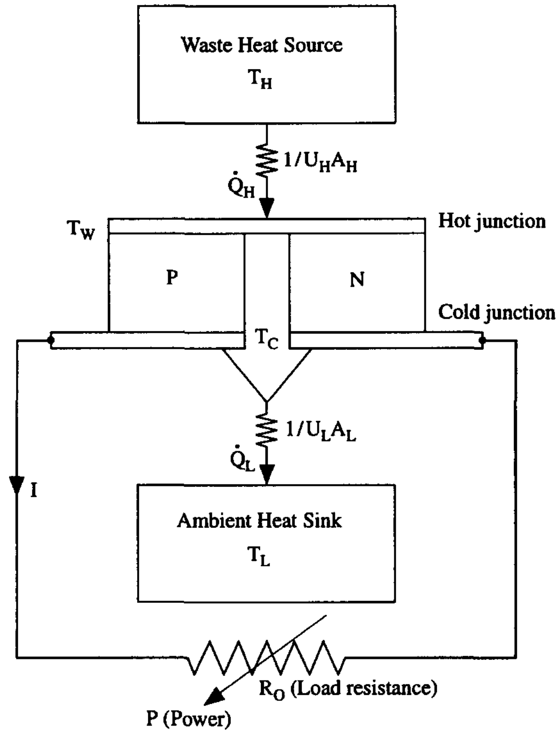

Fig. 3. Carnot heat engine.

each of the two leg materials does not vary with temperature for simplicity. The thermoelectric element is taken to be insulated from its surroundings, except at the junctions.

The thermoelectric generator differs somewhat from the endoreversible waste-heat engine investigated by Wu [3] in a previous paper in that Joulean heat loss and thermal conduction heat flow contribute to the internal irreversibility. The Joulean irreversibility is caused by the semi-conductor electrical resistance, I^2R.

When both legs (n and p) of the generator are considered, the heat is conducted through the two legs in parallel between the same temperature limits and the electric current (I) flows through the two legs in series. The combined conductance of heat transfer (K) and internal electrical resistance (R) for the two legs are

$$K = k_n A_n / L_n + k_p A_p / L_p \tag{1}$$

$$R = \rho_n L_n / A_n + \rho_p L_p / A_p \tag{2}$$

where A and L are the cross-sectional area and length of the thermoelectric element; subscripts n and p refer to the n- and p-semi-conductor material legs, respectively.

When heat conduction, the Joulean heat loss and the energy supply or removal to overcome the Peltier effects are combined for the whole generator arrangement, the rate of heat supply (Q_H), and heat removal (Q_L), useful output power (P) and thermal efficiency (η) are given by Angrist [4]:

$$Q_H = \alpha T_W I + K(T_W - T_C) - 0.5I^2R \tag{3}$$

$$Q_C = \alpha T_C I + K(T_W - T_C) + 0.5I^2R \tag{4}$$

$$P = \alpha(T_W - T_C)I - I^2R \tag{5}$$

$$\eta = P/Q_H. \tag{6}$$

The rates of heat flow from the high temperature waste heat source reservoir to the generator and from the generator to the low temperature ambient heat sink reservoir are

$$Q_H = U_H A_H (T_H - T_W) \tag{7}$$

$$Q_L = U_L A_L (T_C - T_L), \tag{8}$$

where U is the overall heat transfer coefficient including conduction, convection and radiation modes; A is the surface area of the heat exchanger between the generator and its surroundings; subscripts H and L refer to the higher and lower temperature side heat exchangers, respectively.

A heat engine operating between two thermal reservoirs at different temperatures (T_H and T_L) displays its highest efficiency if used in completely reversible conditions ($T_H = T_W$ and $T_C = T_L$) and is known as the Carnot heat engine. Since reversibilities assume infinitely slow heat transfer, the completely reversible Carnot heat engine does not produce any output power. Hence, the reversible Carnot heat engine efficiency is useless for practical engineers. It only gives us an upper bound that is too high to reach for any real heat engine. There is a need to find a power and efficiency bound for a real thermoelectric generator that delivers a positive power.

Consider $U_H A_H$, $U_L A_L$, and all the thermoelectric material properties to be fixed. Also consider the geometry of the thermoelectric couple to be optimized for power output. The output power of the irreversible thermoelectric generator is then a function of T_W, T_c and I. Combining equations (3) and (7) yields

$$T_W = [U_H A_H T_H - K(T_W - T_C) + 0.5I^2R]/(\alpha I + U_H A_H). \tag{9}$$

Similarly, equations (4) and (8) give

$$T_C = [U_L A_L T_L + K(T_W - T_C) + 0.5I^2R]/(U_L A_L - \alpha I). \tag{10}$$

The power output (P) of the thermoelectric generator and the voltage (V) across the load are

$$P = Q_H - Q_L \tag{11}$$

$$V = (T_W - T_C)\alpha - IR. \tag{12}$$

Power generation occurs between open circuit ($I = 0$ and $P = 0$) and short circuit ($V = 0$ and $P = 0$). There is a maximum power between these two extreme conditions. Taking the first partial derivative of P with respect to I and setting it to zero yields

$$I_m = \alpha[(T_W)_m - (T_C)_m]/2R. \tag{13}$$

Substituting I_m into equations (9) and (10) yields

$$(T_W)_m = [U_H A_H T_H - 2RI_m K/\alpha + 0.5(I_m)^2R]/(\alpha I_m + U_H A_H) \tag{14}$$

$$(T_C)_m = [U_L A_L T_L + 2RI_m K/\alpha + 0.5(I_m)^2R]/(U_L A_L - \alpha I_m). \tag{15}$$

Solving equations (13)–(15) simultaneously for the three variables I_m, $(T_W)_m$ and $(T_C)_m$ and substituting them into equation (5), we obtain the maximum power output (P_m) of the waste-heat thermoelectric generator:

$$P_m = \alpha[(T_W)_m - (T_C)_m]I_m - (I_m)^2R. \tag{16}$$

In designing a waste-heat thermoelectric generator for maximum power output, engineers try to attain the given power with minimum volume, minimum weight, or minimum thermoelectric material. To obtain these objectives, a specific power (p), defined by Wu [5] as the power output per unit total heat exchanger surface area, is used in this paper:

$$p = P/(A_H + A_L). \tag{17}$$

The maximum specific power output can then be found by setting power (P) equal to its maximum and dividing by the total heat transfer area.

Notice that the maximum power output and efficiency of the waste-heat thermoelectric generator are linked directly to the temperatures of the heat source and sink. These expressions can serve

as an accurate bound for the best observed performance of existing thermoelectric generators and a guide to the best waste-heat thermoelectric generator design for practical engineers.

NUMERICAL EXAMPLE

As an example, consider a waste-heat thermoelectric generator made of n-type semi-conductor (75% Bi_2Te_3 and 25% Bi_2Se_3) and p-type semi-conductor (25% Bi_2Te_3 and 75% Sb_2Te_3) materials with the following average properties and geometries:

$$\alpha_n = -195 \times 10^{-6}\ \mathrm{V/K},$$

$$\alpha_p = +230 \times 10^{-6}\ \mathrm{V/K},$$

$$\rho_n = 1.35 \times 10^{-3}\ \mathrm{Ohm\text{-}cm},$$

$$\rho_p = 1.75 \times 10^{-3}\ \mathrm{Ohm\text{-}cm},$$

$$k_n = 0.014\ \mathrm{W/[cm(K)]},$$

$$k_p = 0.012\ \mathrm{W/[cm(K)]},$$

$$L_n = 1\ \mathrm{cm},$$

$$L_p = 1\ \mathrm{cm},$$

$$A_n = 1\ \mathrm{cm^2/couple},$$

$$A_p = 1.14\ \mathrm{cm^2/couple},$$

$$R = 2.88 \times 10^{-3}\ \mathrm{Ohm/couple},$$

$$K = 0.0277\ \mathrm{[W/K]/couple},$$

$$\alpha = 425 \times 10^{-6}\ \mathrm{V/K}.$$

Also let the available waste-heat source and the ambient heat sink temperatures be $T_H = 400$ K and $T_L = 300$ K. The hot side and cold side heat exchangers have the following data on a per unit thermoelectric couple basis: $U_H = 0.7$ W/[cm^2(K)], $U_L = 0.7$ W/[cm^2(K)] and $A_H = 0.104$ cm^2. The optimum hot junction temperature [$(T_W)_m$], optimum cold junction temperature [$(T_C)_m$], optimum heat transfer input [$(Q_H)_m$], optimum total heat transfer area [$A_T = (A_H + A_L)$], optimum current (I_m), optimum efficiency (η_m), optimum power output (P_m) and optimum specific power output [$p_m = (P_m/A_T)$] at the maximum power output condition of four classes of waste-heat heat engine are discussed in the following cases.

Case 1. Carnot (both internal and external reversible) heat engine (Fig. 3)

In order to have an externally reversible heat engine, the temperature difference between the warm waste-heat temperature and the hot junction temperature must be infinitely small, that is, $T_H = T_W$. Similarly, $T_C = T_L$. Therefore, the heat exchanger surface areas between the waste-heat thermoelectric generator and its heat source and sink must be infinitely large. The specific power, power output per unit total heat exchanger surface area, produced by the thermoelectric generator, is zero.

Case 2. Endoreversible (internal reversible but external irreversible) heat engine (Fig. 1)

The endoreversible waste-heat thermoelectric generator is an external irreversible but internal reversible Carnot heat engine which is made of two irreversible heat transfer processes and two isentropic processes. The heat addition process is irreversible because heat flows from the high waste-heat temperature reservoir at T_H to the hot junction at temperature T_W across a temperature difference ($T_H - T_W$). Similarly, in the heat rejection irreversible process, the heat flows across the temperature difference ($T_C - T_L$) from the cold junction at a temperature T_C to the ambient heat

sink at a temperature T_L. The power output, hot junction temperature, cold junction temperature, efficiency and specific power output can be calculated [5] from the following equations:

$$C = [(U_H A_H T_H)^{0.5} + (U_L A_L T_L)^{0.5}]/[U_H A_H)^{0.5} + (U_L A_L)^{0.5}] \tag{18}$$

$$(T_W)_m = C[(T_H)^{0.5}] \tag{19}$$

$$(T_C)_m = C[(T_L)^{0.5}] \tag{20}$$

$$(Q_H)_m = U_H A_H (T_H - T_W) \tag{21}$$

$$\eta_m = 1 - [(T_C)_m/(T_W)_m] \tag{22}$$

$$P_m = \eta_m[(Q_H)_m] \tag{23}$$

$$p_m = P_m/A_T = P_m/(A_H + A_L). \tag{24}$$

Case 3. Ideal (external reversible but internal irreversible) thermoelectric generator (Fig. 4)

The ideal waste-heat thermoelectric generator is an external reversible and internal irreversible heat engine. The temperature of the hot junction is equal to the temperature of the waste-heat source reservoir and the temperature of the cold junction temperature is equal to the temperature of the ambient heat sink reservoir, respectively. The internal irreversibility is caused by the Joulean loss and conduction heat transfer. The current, heat transfer input, power output, efficiency and specific power output are calculated by equations (13), (3), (6) and (17), respectively.

Case 4. Real (internal and external irreversible) thermoelectric generator (Fig. 2)

The real waste-heat thermoelectric generator is a heat engine with both external and internal irreversibilities. The external irreversibility is caused by the temperature differences between the hot and cold junctions and the heat source and sink. The internal irreversibility is cause by the Joulean loss and heat conduction. The power output, current, hot junction temperature, cold junction

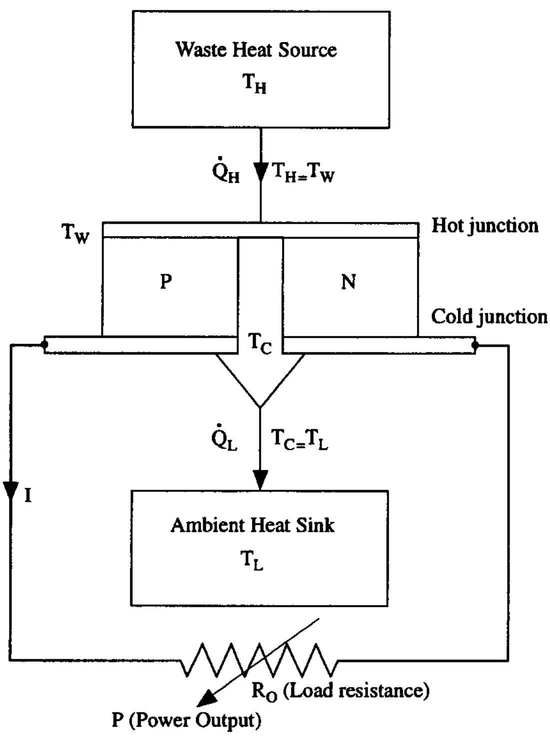

Fig. 4. Ideal waste-heat thermoelectric generator.

Table 1. Comparison of four different waste-heat thermoelectric generators at maximum power

	Generator			
	Carnot (Case 1)	Endoreversible (Case 2)	Ideal (Case 3)	Real (Case 4)
Hot junction temperature (K)	400	373	400	373
Cold junction temperature (K)	300	323	300	323
Heat transfer input (W/couple)	3.95	1.95	3.95	1.95
Heat exhanger total surface area (cm^2/couple)	infinity	0.207	infinity	0.222
Current (amps)	—	—	7.38	3.69
Power output (W/couple)	0.988	0.269	0.157	0.0392
Efficiency (%)	25.0	13.4	3.97	2.01
Specific power output [(W/cm^2)/couple]	0	1.30	0	0.177

temperature, efficiency and specific power are calculated by equations (16), (13), (14), (15), (6) and (17), respectively.

The calculated values of the above four types of waste-heat thermoelectric generators at their maximum power conditions are summarized in Table 1.

CONCLUSION

The concept of a waste-heat thermoelectric generator offers many potential advantages in simplicity, reliability and safety. Its economic competitiveness appears to depend on successful development of new thermoelectric materials and power module designs. The potential for decreased waste-heat thermoelectric generator costs and increased market penetration are promising.

Also, the concept of a completely reversible heat engine has played a major role in the development of the performance of thermoelectric generators. The engineering academic community has used the ideal thermoelectric generator efficiency as an upper bound for external irreversible thermoelectric generators. However, it is a relatively poor guide to the efficiencies of real waste-heat thermoelectric generators. Also, the external reversible ideal waste-heat thermoelectric generator does not generate specific power.

This paper presents a real waste-heat thermoelectric generator model to account for both internal and external irreversibility effects. This approach gives a much more realistic generator specific power and efficiency prediction than does the ideal thermoelectric generator.

REFERENCES

1. C. Wu, R. L. Kiang, V. J. Lopardo and G. N. Karpouzian, Finite-time thermodynamics and endoreversible heat engines. *Int. J. Mech. Engng Eduction* **21,** 337–346 (1993).
2. J. M. Gordon, Generalized power versus efficiency characteristics of heat engines: the thermoelectric generator as an instructive illustration. *Am. J. Physics* **59,** 551–555 (1991).
3. C. Wu and W. H. Schulden, Maximum obtainable specific power of high temperature waste heat engine. *Heat Recovery Systems & CHP* **15,** 13–17 (1995).
4. S. W. Angrist, *Direct Energy Conversion*, 4th Edn. Allyn and Bacon, Boston (1982).
5. C. Wu, Specific power optimization of closed-cycle OTEC plants. *Ocean Engng* **17,** 307–314 (1990).